1. Introduction

This paper analyzes the effect on investment levels of “cancelation rights,” which enable the government (client) to cancel projects before their completion, and of imposing necessary “financial constraints” on projects. The latter are discussed with respect to projects supplying public goods (and services) via a Private Finance Initiative (PFI). PFI is a privatized method of supplying public goods by obtaining private-sector funding and using private-sector technology and managerial knowhow.

Using PFI, the central government and local public entities have created a large social infrastructure. Over the decade from 2010 to 2020, the government aims to expand the scale of these undertakings to more than 12 trillion yen, primarily with such major projects as airports and sewage facilities. The government has established this as a growth area and has made numerous revisions to related statutes. It has incorporated “Opening Public Projects to the Private Sector and Promoting Public Projects Using Private Sector Funds” as Item 14 in its New Growth Strategy.

However, the supply of public goods and services is believed to

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1 The total project amount over the 11-year period from the end of fiscal 1999 through the end of fiscal 2010 equals 4.7 trillion yen and covers such diverse areas as schools, waste processing plants, prisons, hospitals, and libraries.
2 Approved by the Cabinet on June 18, 2010.
be inadequate owing to differences in project contracts and funding methods. That is, because of cancelation rights based on project content and financial constraints based on whether the contractors have their own funding sources for the project or not. The government can exercise its right to cancel the project if the quality of the public goods appears unimproved with respect to the project contract specifications. Financial constraints are problematic when the contractors use internal funding that does not require repayment or when they use external funding from a financial institution that must be repaid. The presence or absence of cancelation rights and financial constraints is feared to indicate the failure to achieve optimal investment levels because projects would not be completed on time or loan contracts would set repayment schedules. In other words, these issues are caused by over-investment or of hold-ups due to under-investment.

Existing research regarding investment in PFI projects includes Oshima (1999), whose study focuses on service transfer fees and the quality of public goods and discusses an incomplete contract model to conduct a quantitative comparison of the effect of the public project method and PFI on the volume of public goods supplied. Mitsui (2005) similarly focuses on the risks and information contained in both methods while Bennett and Iossa (2006) analyze the effect on investment levels depending on whether such public goods are owned by the government or the private sector.

However, while prior studies have analyzed investment levels, almost no analyses have been conducted from the perspective of financial constraints or cancelation rights. With future expansions in the PFI market, the government will be able to monitor financial constraints, which include whether a contractor is able to procure sufficient capital
or whether public goods are being supplied properly. Such monitoring may cause some contract cancelation issues. Furthermore, these factors are simultaneously believed to effect investment levels. Thus, analyzing financial constraints and cancelation rights will not merely identify the mechanisms involved but may also enable examination of whether they reach optimal investment levels or not. Thus, this paper is significant because it goes beyond merely studying existing PFI methods.

Section 2 explains the model, Section 3 contemplates a first best solution and Section 4 provides a detailed analysis of both financial constraints and cancelation rights; finally, Section 5 presents our conclusions.

2. The Model

The multi-period model employed in this study runs from Date0 to Date4. The players are the government, the special purpose companies (SPC) supplying public goods, and the financial institutions providing financing. Each player is risk neutral. While the government aims to maximize social welfare, the other players aim to maximize their expected returns. Moreover, the market interest rate and rate of time preference are both zero.

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3 The model used in this paper is based on Edlin, A.S. and Reichelstein, S. (1996), Seshimo, H. (2003), and Sato and Hosoe (2003).
4 Players providing financing are not necessarily financial institutions. Players having investment capital can also include SPC investment arms and group companies, trading companies, and general contractors.
Figure 1 represents this model’s timeline.

We analyze two cases of financial constraints: when SPC have financial constraints on receiving funding from financial institutions and when they do not. In case of financial constraints being imposed, a financial institution provides funding to the SPC on Date0 for production costs $k$ incurred during the production of public goods and enters into a loan contract $\{k,R\}$ to repay amount $R$ by Date3. A third party verifies the loan contract. With no funds remaining at the SPC, $k$ is invested for the production of public goods on Date2. The government and the SPC enter into a project contract on Date1. If the contract term can be verified as not having cancelation rights, then an executable contract $\{s,t\}$ will be concluded that establishes the contract term $t$ and includes service transfer fees $s$ to be paid by the government to the SPC for the produced public goods. Service transfer fees $s$ are only incurred from Date3 to Date4 and not from Date1 to Date3. The conversion value of an alternative investment opportunity for the SPC is denoted by the exogenous variable $v$. On Date2, the SPC makes a related special investment in the specific PFI project, the cost of which is $i$. Between Date3 and Date4, the SPC’s utility $S$ increases. $S$ is made up of $u$, which is a function of $i$ and is concave, as expressed by $u'(i)>0$, $u''(i)<0$. Here, both $i$ and $u(i)$ are unverifiable. On Date3, the unverifiable conversion value $v \in [v_{min},v_{max}]$ appears in probability distribution $F$, thus leading to a renegotiation of the project contract. However, $F'' \leq 0$. At this point, the SPC generates revenue and the loan is due. If the SPC fails to repay the contracted amount, ownership of the remaining facilities is transferred to the financial institution.
3. First Best

Let us determine the first best solution, which is the investment that maximizes the players’ joint profits. This is the conversion value's distribution $F(v)$ and therefore joint profit $G(i)$ can be found using the following equation:

$$G(i) \equiv \int \max[u(i),v]dF(v) - i$$

Mutual gain is defined as the difference between the larger of either utility or conversion value from the supply of public goods dependent on investment levels, and expected gain with investment costs deducted. This gives us the following:

$$G(i) = \int u(i)dF(v) + \int v dF(v) - i$$

The first best solution is the one that maximizes this equation. Thus, if we assume an interior solution, the first best solution satisfies the following first-order condition:

$$G'(i) = F'u(i^{FB}) - 1 = 0$$

As the conversion value is independent of investment level, the optimal investment level $u(i) - i$ is smaller than the maximum $i$. The following discussion considers this as the benchmark.

4. Second Best

4.1. The Case of No Financial Constraints

Let us analyze three cases with no financial constraints. In 4.1.1, the contract has cancellation rights, the one in 4.1.2 does not, and the one in 4.1.3 has cancellation rights that are applicable after a project is completed but before the contract term ends.

4.1.1. A Contract with Cancellation Rights

When the contract contains cancelation rights, the government can
nullify the project contract with an SPC on Date 3 and thus exercise cancelation rights if the conversion value is greater than the expected return from the public goods. This is possible because neither party made this a point of agreement when renegotiating the contract. On the other hand, if the conversion value is less than the expected return from the public goods, the government will establish service transfer fees with the SPC during the renegotiations. Therefore, if the SPC’s negotiating power is assumed to be $\beta \in (0,1)$, the SPC’s expected return is as follows:

\[
\begin{align*}
0 & \quad \text{if } v \geq u(i) \\
\beta [u(i) + s] & \quad \text{if } v < u(i)
\end{align*}
\]  

Thus, in this case, even though a conversion value occurs when $v \geq u(i)$, the government will pay 0 because this is not a service transfer fee accompanying public goods supply. On the other hand, if $v < u(i)$, the government will renegotiate rather than exercise its cancelation rights. The SPC’s threat point is $v$ while the government’s threat point is 0, thus the negotiating range is $u(i) - v$. This is distributed according to the proportion of negotiating power.

At Date 2 in Equation (2), the SPC’s expected return is decided to be maximized as follows:

\[
S(i) = \int_{u(i)}^{u(i)} \beta [u(i) - v] dF(v) - i
\]  

The first-order condition for maximizing Equation (3) is that the investment level should satisfy Equation (4):

\[
\beta F[u(i^c)]u'(i^c) = 1
\]

We therefore propose the following lemma.

Lemma 1. (Hold-up Problem)

In case the contract imposes no financial constraints on an SPC and gives it no cancelation rights, the related specific investment becomes an
under-investment.

Proof: Based on Equations (1) and (4), $\beta F[u(i_C)]u'(i_C) < 1$ and $i_C < i_{FB}$ are true. QED.

4.1.2. A Contract without Cancelation Rights

When the contract does not contain cancelation rights, the SPC can execute the project contract even if negotiations fail; therefore, the SPC’s threat point is $u(i) + s$ while that of the government is the service transfer fee. The SPC’s returns on Date3 following negotiations are as follows:

$$\begin{align*}
    &\begin{cases}
        u(i) + s + \beta [v - (u(i) + s)] & \text{if } v \geq u(i) \\
        u(i) + s + \beta [u(i) - (u(i) + s)] & \text{if } v < u(i)
    \end{cases} \\
    &\begin{cases}
        (1 - \beta) [u(i) + s] + \beta v & \text{if } v \geq u(i) \\
        u(i) + s & \text{if } v < u(i).
    \end{cases}
\end{align*}$$

In case $v \geq u(i)$, the government would want to nullify the project contract with the SPC but cannot do so because it has no cancelation rights.

The government will simultaneously increase social welfare by not renegotiating the initial contract with the SPC.

In summary,

$$\begin{align*}
    &\begin{cases}
        (1 - \beta) [u(i) + s] + \beta v & \text{if } v \geq u(i) \\
        u(i) + s & \text{if } v < u(i).
    \end{cases}
\end{align*}$$

Based on the above, the SPC will decide to maximize its expected return as follows:

$$S(i) = \int_{-\infty}^{u(i)} [u(i) + s] dF(v) + \int_{u(i)}^{\infty} [(1 - \beta) [u(i) + s] + \beta v] dF(v) - i$$

The first-order condition for maximizing Equation (5) is determined as the investment level required to satisfy the following equation:

$$|F[u(i_{NC})] + (1 - \beta) [1 - F[u(i_{NC})]] u'(i_{NC})| = 1$$

We therefore propose the following lemma.
Lemma 2. (Under-investment)

If the SPC has no financial constraints and no cancelation rights in its contract, the relation-specific investment will be an over-investment.

Proof: Based on Equations (1) and (6), \( F[u(i^{FB})] + (1 - \beta)[1 - F(u(i^{FB}))] \) \( u' (i^{FB}) > 1 \) and \( i^{NC} > i^{FB} \) are true. QED.

4.1.3. Cancelation Rights During the Term of the Project Contract

Let us consider the case where a project contract is concluded on Date1 and the government and the SPC renegotiate the contract on Date3. During period \( t \), while the contract is in force, the SPC makes the investment and earns service transfer fees. Meanwhile, after period \( t \) following Date3, the government has the right to cancel the project contract with the SPC. The SPC’s return for the contract period is the same as in 4.1.2, as follows:

\[
\begin{cases}
(1 - \beta)[u(i) + s] + \beta v & \text{if } v \geq u(i) \\
u(i) + s & \text{if } v < u(i)
\end{cases}
\]

After Date3 and the end of period \( t \), the project is completed and renegotiation begins. The return \( 1 - t \) during the remaining term of the contract is equivalent to the SPC’s expected return in 4.1.1, the case with cancelation rights, as follows:

\[
\begin{cases}
0 & \text{if } v \geq u(i) \\
\beta [u(i) + s] & \text{if } v < u(i)
\end{cases}
\]

Based on these, the SPC decides to maximize

\[
S(i) = t \left\{ \int_{a(i)}^{a(i)^*} [u(i) + s]dF(v) + \int_{a(i)}^{a(i)^*} [(1 - \beta)(u(i) + s) + \beta v]dF(v) \right\} \\
+ (1 - t) \left\{ \int_{a(i)}^{a(i)^*} \beta [u(i) + s]dF(v) - i \right\}
\]
as it is the expected return. The first-order condition for maximization is identified to be the investment level required to satisfy the following equation:
Based on the above, we propose the following theorem.

**Theorem 1.**

*When the SPC has no financial constraints and can enter into a project contract that continues from Date1, considering that \( t^* \) is the project term, then the first best relation-specific investment can be achieved.*

Herein, \( t^* \) indicates \( t^* \equiv F[u(i_{FB})] \). The government and the SPC enter into a project contract that fulfills these conditions.

Proof: Considering \( t = t^* \), the first-order condition for maximization is

\[
\{[\beta+(1-\beta)t]F(u(i))+ (1-\beta)t[1-F(u(i))]\} u'(i) = 1
\]

\( i = i_{FB} \) meets this condition. Therefore, the SPC invests \( i_{FB} \). Furthermore, the government and the SPC establish a contract term on Date1 that maximizes the sum of both player’s returns, which is equivalent to the joint return and independent of the contract term. Thus, the contract is concluded at \( t = t_{FB} \), where the joint return is maximized. QED.

4.2. The Case with Financial Constraints

Here, we consider cases in which the SPC has financial constraints. In 4.2.1, the contract contains cancelation rights while in 4.2.2, it does not.

4.2.1. Loan Contract with Cancellation Rights

When the contract includes cancelation rights, the government can cancel the project contract with the SPC during renegotiations on Date3.

When the conversion value is realized on Date3, the SPC’s return can be expressed as follows:

\[
S^c(v) = v + (1-\beta) \max [u(i) - v, 0]
\]

The second term on the right shows the expected return, which is
calculated as a proportion of the negotiating power during renegotiations when the conversion value is the SPC’s threat point. Figure 2 illustrates $S^c(v)$.

\[
\begin{align*}
S^c(v) & \quad v \\
R & \quad u(i)
\end{align*}
\]

**Figure 2. SPC’s Return after Realizing $v$**

When $S^c(v) < R$, the SPC invests in an alternative investment opportunity rather than in the PFI project. This is $S^c(\hat{v}) = R$, where $\hat{v}$ is the conversion value at which both players are indifferent to investment opportunities. In particular, this is expressed as follows:

\[
\begin{align*}
\hat{v} &= \hat{\bar{v}}(R) \quad \text{if} \quad v \geq u(i) \\
\hat{v} &= \hat{\check{v}}(R, i) \quad \text{if} \quad v < u(i)
\end{align*}
\]

Here, $\hat{\bar{v}}(R)$ and $\hat{\check{v}}(R, i)$ are as follows:

\[
\begin{align*}
\hat{\bar{v}}(R) & \equiv R \\
\hat{\check{v}}(R, i) & \equiv \frac{R - (1 - \beta)u(i)}{\beta}
\end{align*}
\]

If the amount owed as of Date0 is sufficiently large, at $(R \geq u(i))$, where
$v < \bar{v}(R) \equiv R$, the SPC will invest in an alternative investment opportunity instead of the PFI project. In other words, if the SPC’s negotiating power with the financial institution is $\gamma \in (0, 1)$, the SPC’s expected return on Date2 is as follows:

$$S(i) = \int_{u(i)}^{\bar{u}(R)} \gamma[u(i) - v]dF(v)$$

Thus, the first-order condition of maximization is

$$\gamma F[u(i)]u'(i) = 1$$

indicating an under-investment of $i_C < i_{FB}$. Furthermore, $i_C$ is independent of the repayment amount because this amount is large enough for the SPC to decide whether it needs to be invested in an alternative investment opportunity, regardless of its utility for the SPC.

To summarize, when the repayment amount is sufficiently large, even though the conversion value is less than $u(i)$, the SPC cancels the PFI project and the player with which the government continues negotiations is usually a financial institution. When the conversion value exceeds $u(i)$, the SPC will not enter into a contract with the government for the PFI project as long as the difference between the expected return from other investment opportunities and that from the PFI project is positive. Moreover, when the SPC negotiates with the financial institution, the latter takes a $1 - \gamma$ percentage of the difference between the return from the PFI project and the conversion value. Thus, the SPC will under-invest.

Next, let us analyze the case in which the repayment amount agreed to on Date0 is sufficiently small, at $(R < u(i))$. In such a case, the SPC will cancel the project if its investment is $v < \bar{v}(R, i)$. The government, which strives to maximize social welfare, will cancel the contract; thus, the SPC’s expected return will be as follows:

$$S(i) = \int_{\tilde{v}(R, i)}^{u(i)} \gamma[u(i) - v]dF(v) + \int_{\tilde{v}(R, i)}^{\bar{u}(R)} \beta[u(i) - v]dF(v) - i \quad (7)$$
Investment levels are decided such that the first-order condition for maximization satisfies the following equation:

\[
\{ \beta F[u(i^c)] + (\gamma - \beta)F[\tilde{v}(R,i^c)] + \frac{(1 - \beta)(\gamma - \beta)}{\beta^2}F[\tilde{v}(R,i^c)][R - u(i^c)] \} u'(i^c) \tag{8}
\]

To summarize, an under-investment occurs when \( \beta > \gamma \), thus making it desirable to make \( R \) as small as possible. If the minimum repayment amount is small enough, the investment level will be the same as in the case without financial constraints because, given Equation (8), the following is true:

\[
\beta F[u(i^c)] u'(i^c) = 1
\]

Thus, the first-order condition is the same as the case without financial constraints.

On the other hand, under-investment occurs when \( \beta \leq \gamma \). In this case, it is optimal that \( R = u(i^c) \) and investment level is the same as in the case where \( R > u(i) \) because if \( R = u(i^c) \) in Equation (8), then

\[
\gamma F[u(i^c)] u'(i^c) = 1
\]

Thus, the first-order condition is the same as in the case where \( R > u(i) \).

**Theorem 2.**

When the SPC has a contract with financial constraints and cancelation rights, it will under-invest. However, this will also depend on the SPC’s negotiating power vis-a-vis the government and the financial institution; therefore, the optimal service transfer fee will be as follows:

\[
\begin{align*}
R < \beta v_{\min} + (1 - \beta)u(i^c) & \quad \text{if } \beta > \gamma \\
R \geq u(i^c) & \quad \text{if } \beta \leq \gamma
\end{align*}
\]

4.2.2. Loan Contracts in Cases without Cancelation Rights

Let us now consider the project during the period from Date3 to Date4 in a case without cancelation rights and having the efficiency of
an investment protected by a loan contract.

The SPC’s return on Date3 after negotiations with the government is as follows:

\[
\begin{cases} 
(1 - \beta)[u(i) + s] + \beta v & \text{if } v \geq u(i) \\
u(i) + s & \text{if } v < u(i)
\end{cases}
\]

On the other hand, when the project contractor transfers from the SPC to the financial institution, the SPC’s expected return after negotiations in a case without cancelation rights is as follows:

\[
\begin{cases} 
0 & \text{if } v \geq u(i) \\
\gamma[u(i) + s] & \text{if } v < u(i)
\end{cases}
\]

If other investment opportunities exceed the amount to be invested in the PFI project, the government and the SPC will renegotiate. This is shown as \( S^{PC}(v) \) in Figure 3.

![Figure 3. SPC’s Return after Realizing Conversion Value](image)

When \( R \leq s \), the SPC will never cancel the PFI project contract and in
such a case, an over-investment will occur, as in 4.1.2. We will therefore examine the case where \( R > s \).

When no difference exists between canceling the PFI project contract and repaying the amount lent by the financial institution, that is, when \( v = \hat{v} \), the following is satisfied:

\[
S^{PC}(\hat{v}) = R
\]

Solving this equation results in

\[
\hat{v}(R, i; s) = \frac{R + s}{1 - \beta} + u(i).
\]

According to the definition of \( \hat{v} \), when \( v < \hat{v}(R, i; s) \), the SPC will cancel the project contract. Foreseeing this, the SPC’s expected return on Date2 is as follows:

\[
S^{PC}(i) = \int_{-\infty}^{u(i)} \gamma[u(i) - s]dF(v) + \int_{\hat{v}(R, i; s)}^{\infty} \{(1 - \beta)[u(i) + s] + \beta v\}dF(v) - i
\]

The investment level is established so that the first-order condition for maximization satisfies the following:

\[
\gamma F[u(i)] + (1 - \beta)\{1 - F[\hat{v}(R, i; s)]\} - \left\{ \frac{\beta R + s}{1 - \beta} + u(i) \right\} F'[\hat{v}(R, i; s)]u'(i) = 1 \quad (9)
\]

**Theorem 3.**

If the contract has financial constraints but no cancelation rights, and if

\[
(1 - \gamma)F[u(i_{FB})] + (1 - \beta)[1 - F(u(i_{FB}))] + [s - u(i_{FB})]F'[u(i_{FB})] \geq 0, \quad (10)
\]

the repayment amount is at the first best investment level.

Proof: Assuming that \( R = R^{\text{max}} = s + (1 - \beta)[v^{\text{max}} - u(i_{FB})] \), because \( \hat{v} = v^{\text{max}} \). Equation (9) will be \( \gamma F[u(i_{FB})]u'(i_{FB}) < 1 \). This indicates an under-investment.

Next, when \( R = s + \varepsilon (\varepsilon \rightarrow + 0) \), then \( \hat{v} \rightarrow u(i) \), and the first-order condition that satisfies the maximization of investment levels is as follows:

\[
\gamma F[u(i)] + (1 - \beta)[1 - F[u(i)]] + [s - u(i)]F'[u(i)]u'(i) = 1
\]

Based on this condition, we can observe that an over-investment occurs in the case of Equation (10). Furthermore, based on the maximization
theorem, investment continues with regard to \( R \). Continuing to move \( R \) can steer under-investments and over-investments, and therefore, a repayment amount exists that will result in a first best solution based on an intermediate-value theorem. QED.

As in 4.1.2, over-investment occurs when the SPC has no financial constraints. Financial constraints lower investment levels. As such, a loan contract coordinating investment levels under the conditions of Equation (10) will lead to a first best solution. If Equation (10) is rejected, investment levels will change discontinuously under \( R = s \), and an optimal investment level will not be observed. If we assume \( R \leq s \), the SPC will never exercise cancelation rights and over-investment will occur. On the other hand, if we assume that \( R > s \), the contract will be canceled and an under-investment will occur.

From Equation (10), and when \( v^{\text{max}} \) and \( u(i^{FB}) \) are sufficiently small, over-investments can be controlled, thus resulting in a first best solution. Moreover, the larger the negotiating power, or the smaller \( \beta \), the more likely that Equation (10) will achieve a first best solution.

5. Conclusion

This paper compared PFI projects with other investment opportunities to examine the influence of SPC financial constraints and cancelation rights on PFI project investment levels.

The primary conclusions are as follows. When SPC have no financial constraints, they under-invest in cases with cancelation rights and over-invest in cases without cancelation rights. Moreover, optimal investment levels can be reached when entering into a contract for a continuing project. When financial constraints exist, SPC will under-invest when
the contract has cancelation rights. Moreover, SPC may decide not to enter into PFI project contracts owing to their expected returns from alternative investment opportunities. If we assume that financial institutions participate in the negotiations to facilitate the signing of a contract, some of the expected returns will be transferred to the financial institution, and the project will be delayed due to under-investment. Nevertheless, we found that optimal investment levels can be achieved under certain conditions.

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