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Dynamic Monopoly Pricing in a Declining Industry

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1 Introduction

Surprisingly, price setting, which is a fundamental strategy for firms, has not been extensively studied in the existing literature on declining industries.[1] The purpose of this study is to investigate what theory predicts about price dynamics with declining demand. To this end, this paper constructs a dynamic model where a monopoly firm produces an old technology product and its demand declines as a new product appears and spreads among consumers.

In the model, consumers are assumed to be myopic as to which product they buy. That means that, every period, consumers compare utility from the old product and the new product. The utility from the new product is measured by consumer's heterogenous preference to the product and its net surplus, which is a stochastic variable and increases in expectation. Since we assume that consumers never buy the old product once they buy the new one, the number of consumers who buy the new product increases as time passes if the price of the old product is constant. By taking this declining number of consumers into consideration,

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[1] An exception is Ota (2011) that investigates pricing in a duopoly model.

the monopoly firm sets its optimal price so as to maximize the sum of expected discounted profits.

By solving the monopoly firm's dynamic problem numerically, this paper presents simulated price paths and their systematic properties. The price path follows a sharp drop, an adjustment process (price can increase and decrease), and a drop to a steady-state level in this order. This transition is closely related with a transition of the number of consumers who remain in the old product market. We also find that the following factors influence price dynamics: rate of technological advance of new product, distribution of consumers' preference to new product, and their preference to old product. In particular, distribution of consumers' preference to new product alternates the path: uniform distribution leads monotonically declining price path, while (truncated) normal distribution can derives non-monotonic price paths.

The basic source of price dynamics is due to two counteracting motives: raise price to earn more current profit or lower price to keep consumers for future profit. An important feature of declining industry is that the number of consumers who remain in the old product market, which partially determine the profits, is affected not only by the price but also the net surplus of new product. Therefore, when the firm sets the optimal price, it has to expect how large the number of remaining consumers would be. For example, if many consumers are in the old product market but the rate of reduction is high, the firm would lower to price to stop the large reduction. Conversely, if only a few consumers stay in the market but the rate is low, the firm would raise the price. These interactions between the firm's motives and the number of consumers generate the price path.

The price dynamics with a declining demand is nontrivial. In the model, the demand function becomes more price inelastic as the number of consumers decreases.[2] This makes us imagine two counteracting motives for pricing. On the one hand, the monopoly firm wants to set higher prices because it now faces more inelastic demand under which higher price induces higher profit. On the other hand, the firm also has an incentive to keep film prices low, in order to delay the adoption of digital camera. A contribution of this paper is to show systematic properties of these nontrivial price paths and provides new forces deriving the dynamics.

Another contribution is that by studying price competition, we can focus on an interaction between firms and consumers. This interaction is a new aspect in the study of declining industries. Until now, declining demand is taken exogenously and unaffected by firm behavior. However, when firms set price, they can, at least to some extent, control declining demand. For example, consider the situation where consumers choose either an incumbent product or a new product that brings higher utility. If the price of the incumbent product is sufficiently less than the new product, consumers would choose the former, even though the latter delivers more service. This shows the possibility that the incumbent firm could reduce the decline in demand by preventing consumers from adopting the new product.

Existing Literature: Many studies on a declining industry do not analyze price paths of its product. In the industrial organization literature, main theme of a declining industry is the optimal timing of exit.[3] Since the main focus is optimal timing

[2]Ota (2009) empirically shows the demand for photographic film becomes inelastic after the introduction of digital camera.

[3]The seminal papers are Ghemawat and Nalebuff (1985, 1990) and Fudenberg and Tirole (1986).

exit, the literature does not explicitly find price path. For example, Ghemawat and Nalebuff (1985, 1990) assume that price goes down to zero in the long run. This assumption is also found in another strand of literature. From the international trade aspect, researchers investigate endogenous protection policy for declining industries.[4] In the models, the declining industry is defined as an industry where its product's price goes down.

To my best knowledge, this paper is the first to investigate monopoly firm's pricing behavior under a declining demand. There are two exceptions that mention price changes in a declining industry. The first paper is King (1998). That paper adds a capacity constraint to the model of Ghemawat and Nalebuff (1985), and analyzes an optimal timing of exit in a two-period setting. King (1998) demonstrates that 'survivor' firms prepare the failure of a rival by increasing output before the rival exits, that pushes down the market price. Since that paper consider a two-period model, we could not see how the price changes over time as demand declines. Second, Yano, Dei, and Ota (2012, 2016) investigate price change in a declining industry under free entry and exit, and theoretically shows that as demand declines the price rises. The current paper does not allow free entry and exit because our aim is to understand the firm's pricing behavior when the exit is not an easy strategy.

In the next section I present the model, and section 3 provides simulated price paths and their systematic properties. In section 4, we demonstrates comparative statics and see how the price paths are affected by exogenous (technological, consumer-side, firm-side) variables. Section 5 concludes the analysis.

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[4] For example, see Hillman (1982), Long and Vousden (1991), and Choi (2001).

2 Model

2.1 Set up and timing

The model is cast in discrete time and has an infinite time horizon so as to avoid a terminal effect. There are N consumers and two products in this economy. One of them is an existing product and the other is a new product. Since these products are assumed to be substututes, consumers choose either of them in each period. Let N_t be the number of photo film consumers at time t. This implies that the number of consumers who owns the new product is $N - N_t$.

The focus of this paper is a declining industry. To this end, we construct a model where the demand for an existing product declines by the emergence of a new product. The model captures the new product by its net surplus denoted by y_t, and it is an stochastically exogenous variable.[5] As y_t increases, the new product becomes more attractive to consumers leading a decline in the demand for the existing product.

We assume that the net surplus follows a Markov process $y_t = f(y_{t-1}, \epsilon_t)$ where ϵ is an exogenous disturbance, which is *i.i.d.* over time and independent of preceding states and choice variables. Since we consider an infinite time horizon model, the net value of digital camera y_t is bounded by $[\underline{y}, \bar{y}]$ not to explore. Then, the evolution of y_t is given by the following equation:

$$y_t = \max\{\underline{y}, \min\{f(y_{t-1}, \epsilon_t), \bar{y}\}\}$$

In the model, we use the following specification for the process:

$$f(y_{t-1}, \epsilon_t) = \rho y_{t-1} + \epsilon_t \tag{1}$$

[5]This paper treats the net surplus just as an exogenous variable. However, it would be defined as $y_t = x_t - p_t^d$ where x_t is the value of a digital camera and p_t^d is the price of it.

where $\rho > 0$ and $\epsilon \sim N(\mu_\epsilon, \sigma_\epsilon^2)$. The mean of the disturbance is assumed to be strictly positive ($\mu_\epsilon > 0$) so that the net surplus increases in expectation. When $\rho < 1$, we can find the stationary state of y such that $y^* = \min\{E(\epsilon)/(1-\rho), \bar{y}\}$.

The existing product is assumed to made by a monopoly firm. The firm sets a price of the product p_t in the beginning of every period, but it doesn't know the realization of the net surplus y_t at that point of time. Consumers, however, make their decision whether they buy the existing product or the new product after both p_t and y_t are realized.

The optimal policy is obtained by solving backwards. First, given the price of the existing product p_t and the net surplus of the new product y_t, each consumer makes his/her decision. This creates an aggregate demand for the existing product. The monopoly firm sets an optimal price so as to maximize the expect current and future profit based on the aggregated demand function.

2.2 Myopic Consumers

In this section, we develop a demand side model. Ota (2009) finds that the linear inverse demand function becomes steeper in the declining photographic film industry. This price inelastic demand is the source of raising price in the declining industry. Thus, it is important for this paper to describe the process of the change in the demand function. To this end, we focus on a specific class of utility functions and impose on several assumptions on consumer's behavior.

Assume that all consumers have the existing product before the new product is on the market ($N_0 = N$). After the new product is released, consumers buy only one of them. We also assume that once a consumer buys the new product, he/she never buy the old product again.

Consumers are myopic on their decision regarding whether to buy the existing product or the new product. That is, given the price of the existing product p_t and the net surplus of the new product y_t, they compare the current static utility of the new product and that of the existing product, and pick a product that brings higher utility. Let $\phi(y_t, \psi^i)$ be consumer i's utility of having one new product whose net surplus is y_t, and $u(q_t, \theta^i)$ be a utility function of the existing product. Here, θ^i and ψ^i are consumer i's preference parameters on the existing product and the new product , respectively. The associate indirect utility function is $v(p_t, \theta^i)$. Myopic consumers buy a new product if $\phi(y_t, \psi^i) \geq v(p_t, \theta^i)$, and otherwise buy the existing product.

For a further analysis, we specify the utilities from the two goods. The indirect utility from the new product is

$$\phi(y_t, \psi^i) = \psi^i y_t \tag{2}$$

where $\psi^i \in [\underline{\psi}, \bar{\psi}]$ and $0 \leq \underline{\psi} \leq \bar{\psi}$. And the utility function of the existing product is quadratic such that

$$u(q_t^i, \theta^i) = a^i q_t^i - \frac{b^i}{2}(q_t^i)^2 \tag{3}$$

where $\theta^i \equiv (a^i, b^i) > 0$. We choose this functional form because the derived individual demand function is liner, which is employed in an empirical study by Ota (2009):

$$q(p_t, \theta^i) = \arg\max_q [u(q, \theta^i) - p_t q] = \frac{1}{b^i}(a^i - p_t)$$

With this linear demand function, we obtain the associated indirect utility function such as

$$v(p_t, \theta^i) = \frac{1}{2b^i}((a^i)^2 - p_t^2) \tag{4}$$

To make the analysis simpler, the paper assumes that consumers are heterogenous only in their preference to the new product (ψ^i), but they are homogenous in their preference to the existing product, *i.e.,* $\theta^i = \theta \equiv (a, b)$ for all i. Thus, the individual demand function of the existing product is identical to every consumers. Then, the indirect utility function (4) also common to consumers and it becomes $v(p_t, \theta) = \frac{1}{2b}(a^2 - p_t^2)$. Thus we know that if consumer i buys a new product then his/her preference ψ^i must satisfies the following relation:

$$\phi(y_t, \psi^i) \geq v(p_t, \theta) \Rightarrow \psi^i \geq \frac{1}{y_t}\frac{a^2 - p_t^2}{2b} \tag{5}$$

This consumers' decision creates another state variable ψ_t that is a threshold determining who buys the new product ($\psi^i \geq \psi_t$) or the existing product ($\psi^i < \psi_t$) at time t and thus determining the size of market. Since the relation (5) tells us who buys the new product, we can define ψ_t as $\psi_t = \min\left\{\psi^i | \psi^i \geq \frac{1}{y_t}\frac{a^2-p_t^2}{2b}\right\} = \frac{1}{y_t}\frac{a^2-p_t^2}{2b}$. Remember we have assumed that once a consumer buys a new product, the consumer never buy the existing product in the future. Then the equation of motion of the state variable can be rewritten as

$$\psi_t \equiv g(y_t, \psi_{t-1}, p_t) = \min\left\{\psi_{t-1}, \frac{1}{y_t}\frac{a^2 - p_t^2}{2b}\right\} \tag{6}$$

Once we obtain ψ_t, we can calculate the aggregated demand function of photo films at time t. Let $Q(p_t)$ be the aggregated demand and it is

$$Q(\psi_t, p_t) = \int_{\underline{\psi}}^{\psi_t} q(p_t, \theta)[h(\psi)N]d\psi = q(p_t, \theta)N \int_{\underline{\psi}}^{\psi_t} h(\psi)d\psi \equiv q(p_t, \theta)N_t \tag{7}$$

where $h(\cdot)$ is a density function of consumers distribution and N_t is the number of consumers who prefer the existing product at time t.[6] The homogeneity of consumers preference for the existing product makes it easier to derive the aggregated demand function, which is just a summation of the individual demands.

An empirical study by Ota (2009) shows that the demand for old product becomes more price-inelastic as its demand declines. The above aggregated demand satisfies this characteristics. To see this, derive the inverse demand function

$$p_t = a - \frac{b}{N_t} Q(\psi_t, p_t) \tag{8}$$

We can capture a declines in demand by a decline in the number of consumers N_t. Figure 1 illustrates the inverse demand function. As the figure shows, the demand function becomes more price inelastic as N_t becomes smaller.

2.3 Firm

The monopoly firm sets its price before the current net surplus of the new product (y_t) realizes. Thus, the firm doesn't exactly know the effect of p_t on the current number of consumers N_t. The firm can calculate only the expect profit:

$$E_{y_t}\left[(p_t - MC)Q(\psi_t, p_t)\right] \equiv E_{y_t}\pi(\psi_t, p_t)$$

where $\psi_t = g(y_t, \psi_{t-1}, p_t)$. The firm's objective is to maximize the sum of expected discounted profits:

$$\sum_{j=t}^{\infty} E_{y_t}\delta^{j-t}\pi(\psi_j, p_j) \tag{9}$$

[6] In the simulation analysis below, we examine two distribution types for $h(\psi)$: uniform and truncated normal distribution.

where $0 < \delta < 1$ is the firm's discount factor.

Maximization of (9) is accomplished by choice of the optimal sequence of price $\{p_t\}$ for $t = 1, 2, \cdots$. The firm's value function is defined recursively:

$$\begin{aligned} V(y_{t-1}, \psi_{t-1}) &= \max_{p_t}\{E_{y_t}\pi(\psi_t, p_t) + \delta E_{y_t} V(y_t, \psi_t)\} \\ &= \max_{p_t} E_{\epsilon_t}[\pi(g(\psi_{t-1}, f(y_{t-1}, \epsilon_t), p_t), p_t) \\ &\quad +\delta V(f(y_{t-1}, \epsilon_t), g(\psi_{t-1}, f(y_{t-1}, \epsilon_t), p_t))] \end{aligned}$$

By the principal of optimality, the solution to the value function, evaluated at (y_{t-1}, ψ_{t-1}), gives the value of the payoff from the solution to (9) when the initial state is (y_{t-1}, ψ_{t-1}). As the above value function shows, the current price affects the current profit and also the future profit through ψ_t.[7] This is the source of dynamic interaction.

Since this is an infinite time horizon model, we seek a time-independent value for $V(y, \psi)$. On the stationary state, the value function is rewritten as

$$\begin{aligned} V(y, \psi) &= \max_{p} E_{\epsilon}[\pi(g(\psi, f(y, \epsilon), p), p) \\ &\quad +\delta V(f(y, \epsilon), g(\psi, f(y, \epsilon), p))] \end{aligned} \tag{10}$$

We can prove the exitance of the value function (10) because it satisfies the monotonicity condition and $\delta \in (0, 1)$ for applying Blackwell's theorem.

The first order condition for the problem (10) is as follows:

$$\frac{\partial V}{\partial p} = E_\epsilon\left[\frac{\partial \pi}{\partial p} + \delta\frac{\partial V}{\partial p}\right] = E_\epsilon\left[\left(\underbrace{\frac{\partial \pi}{\partial \psi}}_{+}\underbrace{\frac{\partial \psi}{\partial p}}_{-} + \underbrace{\left.\frac{\partial \pi}{\partial p}\right|_{\psi}}_{?}\right) + \delta\underbrace{\frac{\partial V}{\partial \psi}}_{+}\underbrace{\frac{\partial \psi}{\partial p}}_{-}\right] = 0$$

[7]The value function does not explicitly include the number of consumers because it is a function of ψ_t as in (7).

where

$$\left.\frac{\partial \pi}{\partial p}\right|_{\psi} = Q(\psi, p) + (p - MC) \left.\frac{\partial Q}{\partial p}\right|_{\psi} = (p - MC)N\frac{1}{b}\underbrace{(a - p - 1)}_{?}$$

The condition makes it clear that the monopoly firm has two counteracting pricing motives. On the one hand, the firm has an incentive to raise price to earn more current profit. This is described by $\left.\frac{\partial \pi}{\partial p}\right|_{\psi} = (p-MC)N\frac{1}{b}(a-p-1)$. Although the sign of this term is undetermined due to the term of $(a-p-1)$, it could be positive. Then the raise in price increases the current profit. On the other hand, the firm also has an incentive to lower price to keep consumers for future profit. We can see this through two terms of $\frac{\partial \pi}{\partial \psi}\frac{\partial \psi}{\partial p}$ and $\frac{\partial V}{\partial \psi}\frac{\partial \psi}{\partial p}$. These terms shows that an increase in price reduces the number of consumers, and the reduction decreases the profit.

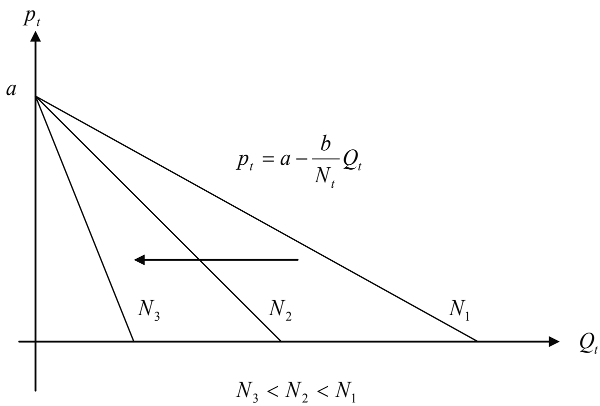
With the counteracting motives, it is difficult to analytically answer questions such as how firms react to declining demand by means of price setting or how the price dynamics are affected by consumers' characteristics. Thus, alternatively, the paper answer these questions by providing a numerical analysis.

3 Price paths with price-inelastic and declining demand

This section presents several simulated price paths based on the model. We take two steps to obtain the price paths. First, we compute the value function (10) by the collocation method. Once we have the value function, then we simulate prices path alone with the motion equations (1) and (6). For the simulation, we need to set an initial value of state variables. They are set at

$$y_0 = 0, \psi_0 = \bar{\psi}$$

Figure 1: An aggregated inverse demand function becomes more price-inelastic as demand declines.



The reason of $\psi_0 = \bar{\psi}$ is that, as equation (5) shows, the consumer who has the highest ψ starts buying the new product. We impose two more assumptions in the simulation. First, the price at time 0 is the static monopoly price ($p_0 = p^m$).[8] The second assumption is a remedy for a subtle problem. When the number of photo film consumer becomes zero ($N_t = 0$), the firm can set any level of price that generates meaninglessly fluctuated price paths. To avoid this, we impose an assumption such that if $N_{t-1} < 1$ then $p_t = p_{t-1}$.[9]

[8]Static monopoly price is given by

$$p^m = \arg\max_{p_t} E_{y_t}\left[(p_t - MC)N_t \frac{1}{b}(a - p_t)\right] = \frac{a + MC}{2}$$

Here price and the market size N_t are assumed to be independent, the static monopoly price does not change even though demand declines.

[9]Notice that we don't impose this restriction when we calculate the value function.

We investigate the simulated price paths when consumers' characteristics is represented by ψ^i, which is uniformly distributed and normally distributed on a bounded support separately. The benchmark distributions are, respectively,

$$\psi \sim U[0,20] \text{ and } \psi \sim N_+(10,4^2) \text{ on } [0,20]$$

where N_+ denotes a truncated normal distribution. In order to keep the difference between uniform distribution and normal distribution clear, we set the same means and domains. The means of both distribution are set at $\mu_\psi = 10$. We set the standard deviation of the normal distribution as $\sigma_\psi = 4$, which means that about 95% population of consumers are distributed in $[\mu_\psi - 2\sigma_\psi, \mu_\psi + 2\sigma_\psi] = [2,18]$. This allows us to fit the normal distribution onto the support $[0,20]$ with less distortion. Another benchmark parameter configuration is

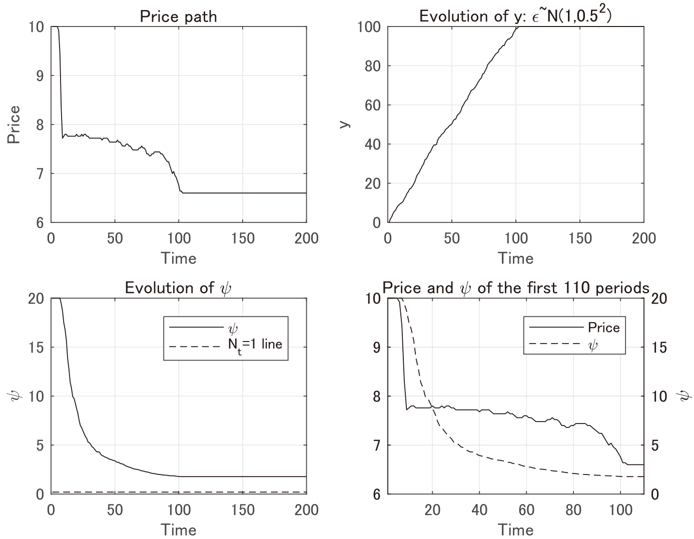
$$\begin{aligned}&(a,b) = (20,1), y \in [0,100], \rho = 1, \epsilon_t \sim N(1, 0.5^2),\\ &N = 100, \delta = 0.9, MC = 0\end{aligned} \tag{11}$$

The discount factor is set at relatively low value of 0.9 because convergence is very slow for high values of δ. The distribution of ϵ_t, the disturbance of the net surplus, is set at $N(\mu_\epsilon, \sigma_\epsilon^2) = N(1, 0.5^2)$. The fluctuation σ_ϵ^2 is smaller than μ_ϵ, which results in a relatively steady increase in y_t. Since price paths are expected to move with y_t, this stable evolution of y_t makes it possible to highlight the firm's counteracting motives in pricing.

3.1 Simulation results: The case of uniform distribution

In this subsection, we characterize the simulated price paths when the consumers' characteristics are uniformly distributed. We mainly

Figure 2: Benchmark case



focus on the benchmark case shown in Figure 2 where price path, evolution of ψ_t and y_t are located at the upper-left, the upper-right, and the lower-left corner, respectively.[10]

We first observe that the price stays at the monopoly price (from period 1 to 4), and then the price sharply drops from the monopoly price. Until the new product is introduced at time 0, we assume that the monopoly firm has set the monopoly price, which is $p_0 = 10$. The net surplus of the new product evolves with random shocks from time 0 and it increases in expectation, but it is quite low in the initial few periods. This implies that consumers obtain higher utility from consuming the existed product. Since

[10]To check the robustness of the observation here, Ota (2010) provides those paths under different parameters such as $\mu_\epsilon = 2$, $\mu_\epsilon = 6$, $\delta = 0.99$, $\psi \sim [2, 18]$, and $\psi \sim [0, 30]$.

nobody buys the new product, the price of existed product can be at the monopoly price.

However, once the net surplus y_t reaches to a certain level, price sharply drops. In this benchmark case, price decreases by about 22.8% just in four periods (period 5 to 9). By comparing with the (d)evolution of ψ_t where high ψ_t means many consumers prefers the old product, we notice that price falls before ψ_t starts falling. This shows that by setting low price, the firm tries to keep more consumers.[11] We can see that this is a reaction of the monopoly firm that sets a dramatically low price in a short period in order to keep the old product attractive to many consumers.

We call these periods Stage I and the price movement of this stage is summarized as below:

Stage I: Price stays at the monopoly price for a few initial times, and then the price sharply drops from the monopoly price.

After the sharp drop, the price stays around a certain level and then decreases gradually until it drops again. First, we explain that price stays around a certain level after the huge price drop. This happens because the monopoly firm's two counteracting motives are equilibrated: raise price to earn more profits now or lower price to keep consumers for the future profit. On the one hand, the firm has a motive to continue lowering price as the net surplus of the new product y_t has been increasing, which makes more consumers switch to the new product. On the other hand, as y_t increases, the number of consumers who buy the existing product decreases and this causes price-inelastic demand

[11] An exception is when y_t evolves quickly $\mu_\epsilon = 6$ where both start falling simultaneously. This would happen because since the rate of technological advance of new product is so high that consumers start switching even if the firm sets a lower price of the old product. But this does not deny the firm's precautionary pricing.

(see equation (8)). This provides an attractive environment for the monopoly firms to execute its market power to raise price.

However, as the value of net surplus becomes bigger, the price decreases gradually until it drops again. This shows that the motive of lowing price is getting stronger than the motive of raising price. We would explain this as follows. Since many consumers has already switched to the new product by this time, there is a small portion of consumers who still prefer the old product. In addition, the net surplus of the new product has been increasing, but its upper bound ($\bar{y}$) is getting closer. Then it would be more profitable in total to lower the price to keep consumers rather than raising the price that leads to less consumers because the number of consumers does not change after y_t reaches the upper bound. In this way, as y_t increases, the motive of lowing price is getting stronger and this leads to the gradual decrease in price.

We call these periods Stage II. In the benchmark case, it lasts about 70 periods (from period 10 to 80), which occupies about 70% of whole time periods where price is adjusted. We see that in a large portion of time, price is gradually adjusted with the evolution of the net surplus of new product. The price movement of this stage is summarized as below:

Stage II : The price stays around a certain level and then decreases gradually. This stage lasts longer than Stage I.

In this simulation analysis, we set an upper bound on the net surplus of the new product, $y_t < \bar{y}$. Once the net surplus reaches the upper bound, the value of new product does not improve. Since there is no stochastic process after y_t reaches $\bar{y}$, price dynamics also vanishes. We call periods after $y_t = \bar{y}$ a steady state.

As a final reaction of the firm, the price drops sharply to the steady state level. The firm starts lowering price several periods before y_t hits its upper bound. This shows that the motive of lowering price overwhelms the motive of raising price. This is a difference from Stage II where the counteracting motives are almost equilibrated. A key observation to clear this difference is the steady state. Once y_t reaches its upper bound, there are no dynamics in price and no change in the number of consumers who buy the old product. The firm will need to operate its business with the remaining consumers. Thus in the long run, lowering price to retain more consumers would be a better strategy than rasing price.

We call these periods Stage III. In the benchmark case, the price drops by 10% (7.4 to 6.6, which is a steady-state level price) in 24 periods from period 80 to period 103. The price movement of this stage is summarized as below:

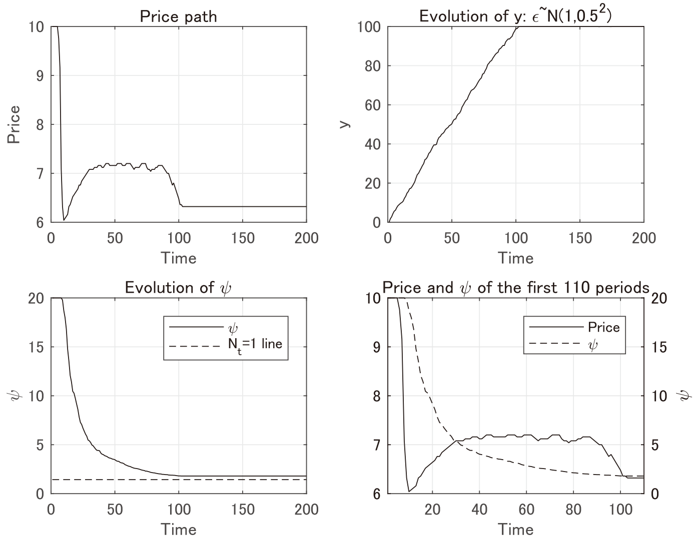
Stage III: The price drops sharply to the steady state level. The steady state is attained when the net surplus y_t reaches its upper bound.

3.2 Simulation results: The case of truncated normal distribution

How does the simulated price path depend on the distribution type of consumers' characteristics? In this subsection, we investigate the price paths when ψ is normally distributed on a bounded support. Different from uniform distribution, normal distribution is unimodal. We set the case of Figure 3 where $\psi \sim N_+(10, 4^2)$ on $[0, 20]$ with (11) as a benchmark of normal distribution cases.[12]

[12] See Ota (2010) for the robustness of the observations in this case. That paper provides alternatives cases such as $\mu_\epsilon = 2$, $\mu_\epsilon = 6$, $\delta = 0.99$, $\psi \sim$

Figure 3: $\psi \sim N(10, 4^2)$ on $[0, 20]$ and $\epsilon \sim N(1, 0.5^2)$



The simulated price paths under the normal distribution of consumers' characteristics are similar to those under uniform distribution except Stage II. We can observe that price increases after the initial sharp drop while the price slightly decreases after a stable path in the uniform distribution cases.

Stage II: When consumers' characteristics ψ is normally distributed, the price path can be in up rate after the initial price drop.

Remember that the monopoly firm sets its price from two counteracting motives: raise price to earn more profits now or lower price to keep consumers for future profit. Simulation results show that the former motive is stronger than the latter in

$N(10, 3^2)$, and $\psi \sim N(10, 5^2)$.

the normal distribution cases while they are equilibrated in the uniform distribution cases. This difference comes from the shape of the normal distribution. The transition of ψ_t shows that only a few consumers prefer the old product after the initial price drop, and these consumers are on the left tail (lower ψ) of the distribution. Since it is a truncated normal distribution, the density on the left tail decreases as the ψ is lower. Thus, even if the firm raises the price that induces a lower ψ_t, this induces a few remaining consumers to switch to the new product. This strengthens the motives for raising the price.

We can summarize the common observation including both types of distribution of consumers' characteristics as follows:

Result: The price path follows a sharp drop, an adjustment process (price can increase and decrease depending on the distribution of consumer's type), and a drop to a steady-state level in this order. This transition is closely related with a transition of the number of remaining consumers.

4 Comparative statics

This section examines comparative statics of the price paths generated under each distribution of ψ. We investigate the following questions:

1. What happens when the rate of technological advance increases? ($y_{t+1} = \rho y_t + \epsilon_t$)
   (a) Change the mean of $\epsilon_t \sim N(\mu_\epsilon, \sigma_\epsilon^2)$
   (b) Change ρ: Spill-over effect and depreciation of previous net surplus.

2. What happens depending on the preferences for the old product?

3. What happens depending on the distribution on consumer types?

   (a) Change the range of domain of distribution: Narrower distribution means more homogeneous consumers.

   (b) Change the mean of the distribution: Larger mean means consumers appreciate new product more.

4. What happens as the discount factor increases so a firm is more patient?

The results of comparative statics are summarized in Table 1.

Figures 4 - 9 show the comparative statics of price paths. In each figure, I put the results of the uniform distribution case in the left column and those for the normal distribution case in the right column.

Rate of technological advance of new product: The net surplus of the new product can be thought as the value of the product, and it is a stochastic process: $y_t = \rho y_{t-1} + \epsilon_t$. The disturbance $\epsilon \sim N(\mu_\epsilon, \sigma_\epsilon^2)$ captures exogenous technological advances, and in particular μ_ϵ represents the average rate of technological advance. Here by changing μ_ϵ, we see what happens to the price paths when the average rate of technological advance increases. Figure 4 shows paths of price, ψ_t and N_t under $\mu_\epsilon = 1, 2, 4$ for each distribution of ψ^i: uniform distribution and truncated normal distribution.

There are two observations in this comparative statics. First, as the rate of technological advance increases, time periods of each phase get shorter. Second, for all cases, price drops to the same

Table 1: Summary of comparative statics

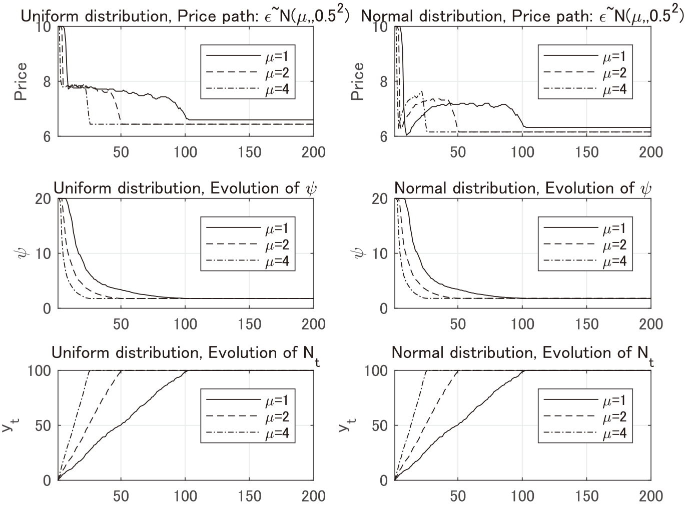
Parameter	Effects on price path
Tech. advance	
μ_ε, ρ	As μ_ε or ρ increase, (i) Each stage becomes shorter; (ii) Shape of price path does not change.
Consumer's side	
a, $[\underline{\psi}, \overline{\psi}]$, μ_ψ	As a increases, (i) Initial price drop becomes larger. When μ_ψ increases or $[\underline{\psi}, \overline{\psi}]$ gets narrower, it holds that (i) and (ii) Price declines (increases) more sharply after the initial drop when ψ is uniformly (normally) distributed.
Firm's side	
δ	As δ increases (the firm becomes more patient), the firm sets lower price in stage III.

level in Stage I. These observations reveal that the firm corresponds to the rate of technological advance more quickly as the rate is higher, but it does not change the optimal policy on each pair of states, y_t and ψ_t.

Alternatively, we see an increase in the rate of technological advance by setting ρ more than 1. Different from changing the mean of ϵ_t, ρ affects the current net surplus through the previous period's net surplus. Thus we can interpret that when $\rho > 1$, the technological advance has a spill-over effect on the net surplus. We set $\rho = 1, 1.01, 1.02$ and compare the price paths.

Figure 5 shows the price paths of this comparative statics.

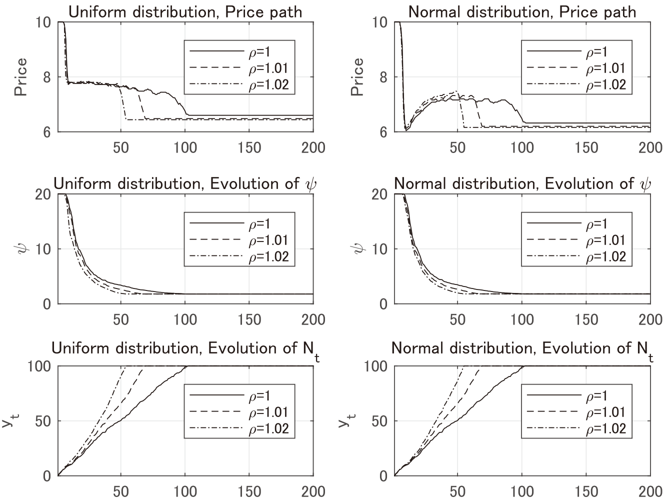
Figure 4: Comparative statics: Rate of technical advance of new product: μ



Observations are the same as ones in the previous case: (i) as the rate of technological advance increases, time periods of each stage get shorter, and (ii) for all cases, price drops to the almost same level in Stage I. The firm responds to the rate of technological advance more quickly as ρ is higher, but it does not change the optimal policy on each pair of states, y_t and ψ_t. From these comparative statics, we find that the rate of technological advances does not change the optimal policy. Thus, the factors that may change the optimal policy would be something that are related with consumers' or the firm's characteristics. In below, we investigate comparative statics of such factors.

Preference to the old product: Consumer's utility function for old product is represented by $u(q_t) = aq_t - \frac{b}{2}(q_t)^2$. Thus the

Figure 5: Comparative statics: Rate of technical advance of new product: ρ

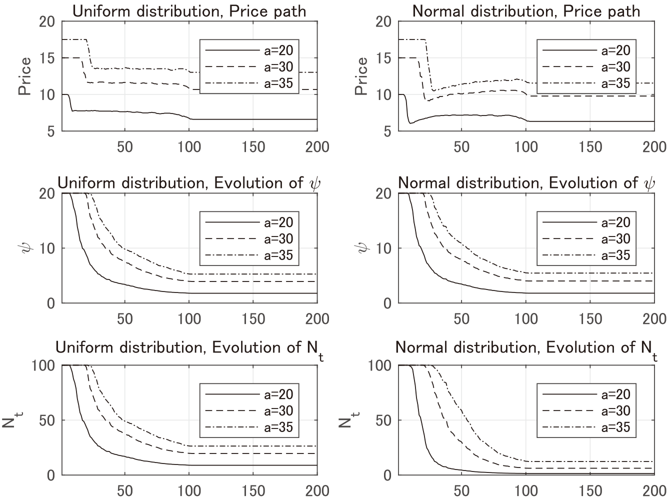


parameter a captures a positive preference to the old product. Here we investigate what happens to price paths when consumers prefer the old product more. Figure 6 compares the price paths under $a \in \{20, 30, 35\}$.

We observe three findings. First, we see that it takes a longer time until the initial price drop as a increases. This finding is common to the distribution type of consumers' characteristics. For example, when ψ is uniformly distributed, price starts falling at period 6 under the case of $a = 20$. It is period 16 and 21 when $a = 30, 35$, respectively. This is intuitive because consumers prefer the old product more as a increases, and begin to switch to the new product with higher y_t.

The second finding is on the price path of the uniform dis-

Figure 6: Comparative statics: Preference to the old product



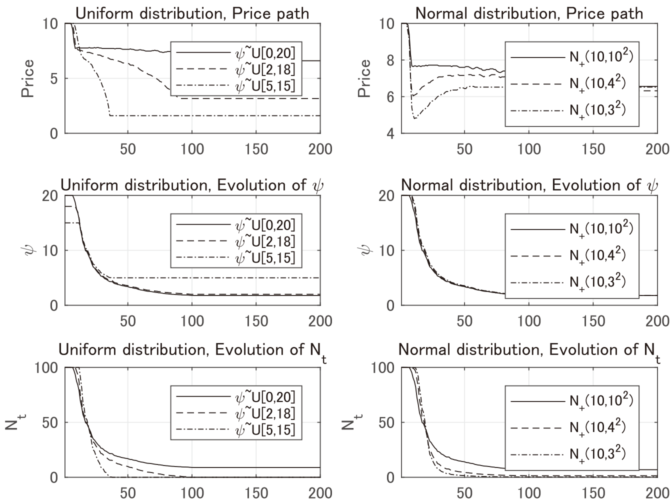
tribution cases. It seems that price drops more in Stage I as a increases. In the uniform distribution cases, price falls by 2.28 (from 10 to 7.72) when $a = 20$, and it is 3.18 and 3.85 when $a = 30, 35$, respectively. However, when we take the drop rate measured by the value of price fell divided by the initial price, it is about 22% for all cases above.

The third finding is seen in the normal distribution cases. It seems that as a increases, it is less obvious to find the price increase in Stage II. We explained that since consumers who prefer the old product are on the left tail of normal distribution by that stage, the price increase does not induce the remaining consumers to switch to the new product. This strengthens the motives of raising the price rather than those of lowering price for the future profit. However, when a increases, there are more consumers who prefer the old product. For example, at period 50, there are more than half of all consumers when $a = 35$ while there are less one quarter when $a = 20$. Since consumers are normally distributed, the density is the highest at the mean. This implies that a price increase would make a lot of consumers switch to new product. Thus, we guess that the firm's motive for raising price in Stage II becomes weaker as a increases.

Distribution of consumers' type: Next, we investigate how the price paths change depending on distributions on consumer characteristics. First, we compare price paths by narrowing the support of the distributions but keeping its mean. A narrow support of the distribution means that consumers are more homogenous. (Consumers are perfectly homogenous in their preference for photo films.) Second, we compare price paths by changing the means of distributions but keeping their supports. Larger mean means that consumers appreciate the new product more.

Figure 7 compares simulated price paths when narrowing the support of the distributions but keeping its mean the same. First, focus on the uniform distribution cases where $\psi \sim U[0, 20]$, $U[2, 18]$, $U[5, 15]$. We find two features in the price paths. As consumers are more homogenous, (i) the initial price drop is bigger and (ii) after the initial price drop, the price paths decline more sharply. These phenomena are explained by the difference in probability density functions. As the supports becomes narrower in uniform distribution, their density becomes larger at each point of ψ in the support. This implies that a marginal decrease of ψ loses more consumers. Through (6), price affects the state variable ψ_t representing the cut off point for who buys the new product. Thus, a higher price induces more consumers to switch to the new product as the support is narrower. This weakens the

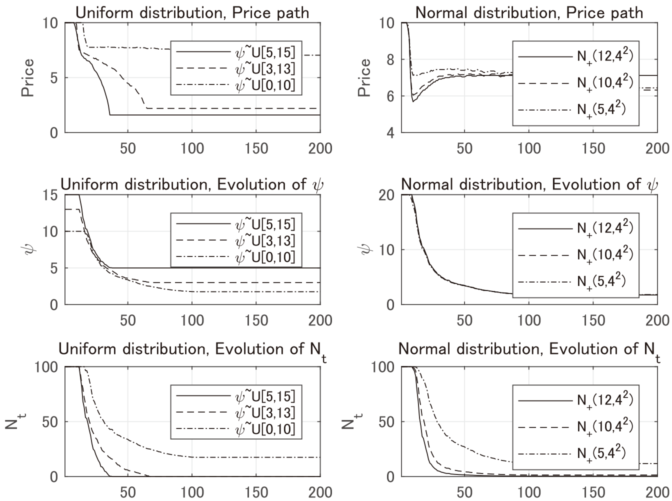
Figure 7: Comparative statics: Narrowing support of distributions



firm's motive to raise price, and strengthens the motive to lower price to keep consumers. Thus, as consumers are more homogenous represented by a narrower support, the firm tends to lower price.

For the normal distribution case, it is less obvious to find the price increase in Stage II as the support of the distribution becomes narrower. This can be also explained by the same logic above. As the support becomes narrower, the associated probability density function takes a larger value at each point of ψ in the support. Since this implies that a marginal decrease of ψ loses more consumers and a higher price lowers ψ_t through (6), the firm weakens its motive to raise the price. In the benchmark case, the price increases in Stage II when consumers' characteris-

Figure 8: Comparative statics: Change the mean of distributions



tics are normally distributed, and this is due to a stronger motive for rasing price. However, as the support of the distribution becomes narrower, the two counteracting motives are equilibrated by the weakening motive for raising price.

Here we compare price paths by changing the means of distributions but keeping their supports. In this comparative static, we see how the price paths change if consumers prefer the new product more. For the uniform distribution cases, we investigate price paths under $\psi \sim U[5,15], U[3,13], [0,10]$ where each mean is 10, 8 and 5, respectively but the range of the support is common to 10. For the truncated normal distribution cases, we investigate the cases where $\psi \sim N(12,4^2), N(10,4^2), N(5,4^2)$ supported by $\psi \in [0,20]$.

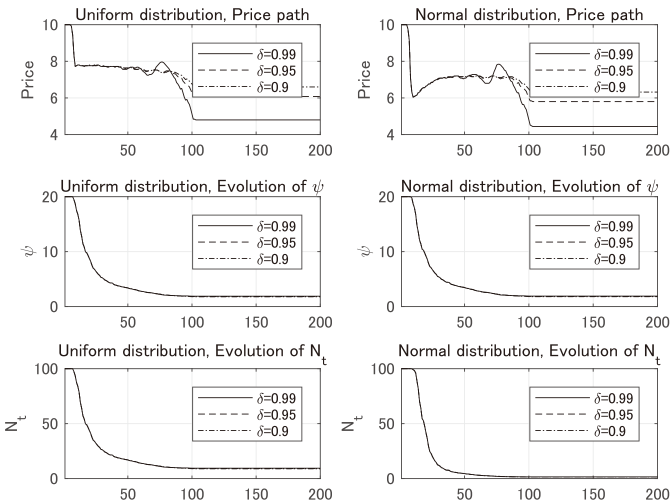
To do this, we change the mean of each distribution from their benchmark case. Figure 8 shows simulated price paths and associated paths of ψ_t. We find: as the mean of consumer's characteristics becomes larger, *i.e.,* more consumers prefer the new product, (i) the initial price drop is bigger and (ii) after the initial price drop, the price paths decline (increase) more sharply when ψ^i is uniformly (normally) distributed.

First, consider the uniform distribution cases. Since their support range, which is 10, is the same, their probability density function is the same, too. Thus, one unit of decrease in ψ loses the same number of consumers. In the last comparative statics of narrowing the support, this effect of a decrease of ψ on N_t is important to explain the price decline. However, even with the same magnitude of the effect, price decreases more sharply as consumers on the whole prefer the new product.

By looking at the middle figures of Figure 8, we notice that the transition path of ψ_t, which is a state variable representing the cutoff point who buys the new product, does not vary by the distributions. This implies that at each time period, there are less consumers who prefer the old product for the distribution having a larger mean. Then we can guess that the number of consumers at each point of time is another important factor to set price. With less consumers the motive for lowering price is stronger than the motive for rasing price for the future profits, while the latter is stronger than the former with more consumers.

Similarly, in the normal distribution cases, the transitions of the state variable ψ_t do not vary by the distribution. Thus, at each time period, there are fewer consumers who prefer the old product for the distribution having a larger mean. In the benchmark case of the normal distribution, we saw the price increase in Stage II. However, in the normal distribution with small mean, the

Figure 9: Comparative statics: Discount factor of the firm



price declines. When the distribution has a smaller mean, there remains more consumers who prefer the old product. With more consumers, the motive for raising price overwhelms the motive for lowering price.

Discount factor of the firm: Finally, we consider whether the firm's characteristics affect the price path. By changing its discount factor, we see what happens to the price path as the firm becomes more patient. Figure 9 shows the price paths with $\delta = 0.9, 0.95, 0.99$. We obtain two findings common to both distribution types: (i) the discount factor has little effects on the initial price path, but (ii) steady-state price is in lower level as the firm is more patient.

When the firm is more patient, it appreciates the future profits more. Thus, we can expect that the firm sets lower price to keep as many consumers as possible. This explains the second finding: the more patient the firm is, the lower the firm sets price at the steady state level. However, as the first finding shows that the firm's discount factor affects the price path in the later periods when the upper bound of the net surplus is closing. In the early periods after the introduction of new product, the price is set according to the evolution of y_t but not the discount factor δ.

5 Conclusion

This paper constructs a dynamic model where a monopoly firm produces an old technology product and its demand declines as a new product appears and spreads among consumers. The paper presents simulated price paths and their systematic properties.

Through comparative statics, this paper demonstrates that distribution type of consumer's characteristics is a critical factor in alternating price paths. Under an assumption that myopic consumers never buy an old product once they buy the new one, the number of current consumers and the rate of reduction are important points for price setting. The distribution of consumer's characteristics affects these points.

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