Pricing an Existing Product while Producing a New Substitute

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1 Introduction

In the modern economy, firms frequently invent and introduce technologically new products into markets, such as digital cameras, smartphones, memory devices (FD, CD, DVD, USB, and Cloud), and so on. In some cases, firms release new products while still producing substitutable products. For example, Kodak produced both digital cameras and classical cameras, and Apple sells old and new iPads simultaneously. This action seems nonsensical because newly introduced products generally take demand away from substitutable existing products. Therefore, how can we rationalize the firm's behavior?

To answer this question, we focus on pricing both old and new products. Essentially, our research question is as follows: how do firms set a price for such existing products when they sell new products? The answer is not obvious because there are two opposite motives for pricing. One is to lower the price of the existing product. This may maintain demand for the old product but prevent the proliferation of the new product. The other motive is to raise price instead, resulting in the loss of demand for the old product and the proliferation of the new product. In this study, we examine how these motives interact theoretically.

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To achieve this, we construct a dynamic monopoly pricing model in which only the firm can produce and sell both old and new products. We focus on a monopoly because only one firm introduces innovative new products, at least in its early stages. The difference between the old and new products is durability and consumers' marginal willingness to pay. Using the model, we demonstrate that the monopoly firm will lower the price of the old product immediately after releasing the new product and then increase the price, while it will first set the price of the new product higher and then lower the price. Although it would be reasonable to set a higher price for the new product because it is durable, notably, the price of the old product rises, that is, the firm sets a lower price in the first period. This phenomenon can be explained by a market expansion effect. A lower price makes the old product more attractive to consumers. Some consumers buy the old in the first period, and they will purchase either the old or the new product in the second period. If the firm sets a higher price for the old product in the first period, many consumers buy the new product in this period and buy nothing in the second period. Lowering the price of the old product in the first period allows consumers to purchase in both periods, which expands the market.

In the literature on declining markets, some studies examine the price paths of the old product, which the new product replaces. For example, Ota (2011, 2019) investigate that firms counteract pricing motives by simulation; they lower prices to delay the adoption of a new product and raise prices to exploit price-insensitive consumers. Yano, Dei, and Ota (2012, 2017) demonstrate that the price of a product facing declining demand rises under Cournot competition with free entry and exit. However,

in these studies, new products and their values are given exogenously. That is, firms could not control the proliferation of new products directly. However, this study contributes to the literature by allowing the firm to control the proliferation of both new and existing products to maximize its profits.

This paper is organized as follows. In the next section, we describe the market structure in which a monopoly firm can produce both old and new products and investigate its pricing behavior. Section 3 discusses the result and proposes the market expansion effect as a possible explanation for the result. Section 4 contains our concluding remarks.

2 Model

The model is cast in discrete time and has two periods.[1] There is only one firm in this economy, which produces two goods simultaneously: an old product and a new one. This study distinguishes these products by their durability. We assume that while the new product maintains its performance for two periods, the old product could do so for only one period. This difference comes from the following observations. In general, a new product's quality is superior to the old product. Once consumers purchase the new product, they usually never buy an old product again. However, a consumer owning the old product but not the new one may face a choice between them in every period, because the new product could potentially increase the consumer's utility. In other words, the introduction of a new product would make the old product

[1] In the long run, a new product will completely replace an old product because of its technical superiority. Thus, pricing the old product in the near future would be more important than pricing it farther in the future. A finite time horizon model could reflect such an idea and keep pricing the old product still important for the firm.

attractive for only one period. Therefore, we could distinguish the products by their durability; consumers utilize the new product for two periods, but they utilize the old product for one period only.

We now explain the consumers' behavior. At the beginning of the first period, every consumer owns an old product. Then they choose the new product or the old one in the first period, when a firm introduces a new product into the market. If consumers purchase the new product in the first period, they do not buy anything in the second period. However, if they purchase the old product, they face the same choice in the second period.

This study assumes that consumers are heterogeneous in preference to the new product.[2] This assumption means that consumers evaluate the new product differently; some evaluate the new product higher than do others. To represent the above idea, we model the marginal willingness to pay for the new product as follows:

$$\text{marginal willingness to pay} = b - ax \tag{1}$$

where $a, b > 0$ and x is the number of consumers who purchase the new product. This means that the first consumer purchasing the new product receives utility of b because nobody purchases the good before it, *i.e.,* $x = 0$. A consumer evaluating the new product second has $b - a$ of marginal willingness to pay. In this way, every consumer has different marginal willingness to pay for the new product, which represents the heterogeneity.

We can interpret equation (1) as an inverse demand function for the new product.[3] Since every consumer purchases only one unit of either of the products, the number of consumers pur-

[2]Preference to the old product is homogeneous among consumers.

[3]See Coase (1972).

chasing the new product x is equal to the number of the product sold. From this interpretation, we know that the total number of consumers is b/a. The last person, *i.e.,* b/ath consumer, receives zero utility from the new product.

To highlight the heterogeneity in the preference for the new product, we assume that consumers are homogeneous in their preference for the old product. They receive α from purchasing an old product. The α is the marginal willingness to pay, and it is common to all consumers. We put $b > \alpha$ as an assumption. This allows at least one consumer that prefers the new product to the old intrinsically.

On the production side, this study assumes that there is only one firm in the economy and this monopoly firm produces old and new products simultaneously.[4] Then, the firm's strategy is to determine the optimal quantity and the associated price $(q_t^i, p_t^i, t = 1, 2 \text{ and } j = new, old)$ to maximize the sum of profits of the two periods. Since the time horizon is finite, we could solve the problem backwards.

Given the production of a new good in period 1, x_1^{new}, the problem in the second period $(t = 2)$ is defined as follows:

$$\begin{aligned} & R_2 \equiv \max p_2^{new} x_2^{new} + p_2^{old} x_2^{old} \\ \text{subject to} \quad & x_2^{new} + x_2^{old} = \frac{b}{a} - x_1^{new} \qquad (2) \\ & b - a\left(x_1^{new} + x_2^{new}\right) - p_2^{new} = \alpha - p_2^{old} \qquad (3) \end{aligned}$$

Equation (2) shows the number of consumers who can purchase the new or old product in the second period. Since x_1^{new} con-

[4]The monopoly assumption could highlight the effect of a newly introduced product on the existing product. Although investigating oligopolistic cases is interesting, these cases generate strategic interactions between firms, which makes the effect of the new product on the price of the existing product unclear.

sumers have already bought the new product in the first period, they do not buy anything in the second. Equation (3) represents the relationship between the prices of the old and new products, p_2^{old} and p_2^{new}. Since the old product and the new one are substitutable, consumers determine their purchasing decision by the net surplus. If the net surplus from buying a new product is larger than that from an old one, consumers buy the new product. However, consumers are heterogeneous in their preference for the new product; then, a consumer exists for whom it is equivalent to purchase the new product and the old one. Such a consumer is the last person who purchases the new product and determines the number of the new product sold, x_2^{new}. Therefore, given x_1^{new}, it must hold that the net surplus from buying the new product $(b - a\left(x_1^{new} + x_2^{new}\right) - p_2^{new})$ is equal to the net surplus from obtaining the old one $(\alpha - p_2^{old})$, which is equation (3).

We obtain the following solutions for the problem:

$$\begin{array}{ll} p_2^{new} = \frac{b - a x_1^{new} + \alpha}{2}, & p_2^{old} = \alpha \\ x_2^{new} = \frac{b - a x_1^{new} - \alpha}{2a}, & x_2^{old} = \frac{b - a x_1^{new} + \alpha}{2a} \end{array} \tag{4}$$

Note that the price of old product p_2^{old} is equal to the constant marginal willingness to pay for it, α, because the firm is a monopoly firm. The joint profit in the second period is $R_2 = \frac{\left(b - a x_1^{new} + \alpha\right)^2}{4a}$.

Given the profit in the second period, the firm sets the quantity in the first period to maximize the sum of profits for the

two periods. The monopoly firm's problem is as follows[5]:

$$\begin{aligned}
\max \quad & p_1^{new}x_1^{new} + p_1^{old}x_1^{old} + R_2 \\
\text{s.t.} \quad & x_1^{new} + x_1^{old} = \frac{b}{a} && (5) \\
& \frac{1}{2}\left\{2\left(b - ax_1^{new}\right) - p_1^{new}\right\} \\
& \geq \max\left\{\left(b - ax_1^{new}\right) - p_2^{new}, \alpha - p_2^{old}\right\} && (6) \\
& 2\left(b - ax_1^{new}\right) - p_1^{new} \\
& = \alpha - p_1^{old} + \max\left\{\left(b - ax_1^{new}\right) - p_2^{new}, \alpha - p_2^{old}\right\} && (7)
\end{aligned}$$

Equations (5) corresponds to (2). Equation (6) is an assumption on the marginal consumer of the first period. The left hand side is the *average* net surplus that the marginal consumer receives over time and the right hand side is the maximum net surplus of the consumer next to the marginal consumer in the second period. The assumption means that in the second period, the marginal consumer obtains at least a larger net surplus than does the next consumer. Mathematically, equation (6) defines the relationship between p_1^{new} and x_1^{new}.

We add some comments on equation (7), which define the relationship between p_1^{new} and p_1^{old}. Importantly, consumers now take decisions by considering two periods. The left-hand side in equation (7) represents the net surplus for the last consumer who buys a new product in the first period. Since the new product is a durable good, its attractiveness continues in the following period. Therefore, we must double the utility from the new product: $2\left(b - ax_1^{new}\right)$. The right-hand side in equation (7) represents the net surplus when the consumer purchases the old product in the first period. Again, this consumer must decide in

[5]Here, for simplicity, we ignore a discount of the future net surplus.

the second period; switch to the new product or repurchase the old one. This is reflected by $\max\left\{(b-ax_1^{new})-p_2^{nw}, \alpha-p_2^{old}\right\}$. However, it can be simplified. From equation (3), it holds that $b-ax_1^{new}-p_2^{new} \geq \alpha-p_2^{old}$ if $x_2^{new} \geq 0$, which is true by definition.[6] Therefore, the equation (3) is rewritten as $2\left(b-ax_1^{new}\right)-p_1^{new} = \alpha-p_1^{old}+\left(b-ax_1^{new}\right)-p_2^{new}$. The solutions of the problem are as follows:

$$\begin{array}{ll} p_1^{new}=\frac{1}{5}\left(2b+7\alpha\right), & p_1^{old}=\frac{1}{10}\left(-2b+13\alpha\right) \\ x_1^{new}=\frac{1}{5a}\left(3b-2\alpha\right), & x_1^{old}=\frac{1}{5a}\left(2b+2\alpha\right) \end{array} \tag{8}$$

3 Discussion

Plugging the optimal x_1^{new} shown in (8) into (4), we represent these second-period variables only by parameters. Table 1 summaries the theoretical results and Table 2 provides a numerical example.

	Period 1	Period 2
Price of new product (p^{new})	$\frac{1}{5}\left(2b+7\alpha\right)$	$\frac{1}{10}\left(2b+7\alpha\right)$
Price of old product (p^{old})	$\frac{1}{10}\left(-2b+13\alpha\right)$	α
Quantity of new product (x^{new})	$\frac{1}{5a}\left(3b-2\alpha\right)$	$\frac{1}{10a}\left(2b-3\alpha\right)$
Quantity of old product (x^{old})	$\frac{1}{5a}\left(2b+2\alpha\right)$	$\frac{1}{10a}\left(2b+7\alpha\right)$

Table 1: Summary of Optimal Prices and Quantities

Restricting the parameters by $\frac{13}{2}\alpha \geq b \geq \frac{3}{2}\alpha$, we can guarantee that all prices and quantities are positive. Remember that we originally assume $b > \alpha$ to ensure that there is at least one consumer who prefers the new product to the old one. Thus, our restriction implies that the original assumption is insufficient

[6] We apply this relationship to Equation (6). Thus, it becomes $\frac{1}{2}\left\{2\left(b-ax_1^{new}\right)-p_1^{new}\right\} \geq \left(b-ax_1^{new}\right)-p_2^{new}$.

	Period 1	Period 2
p^{new}	11.0	5.5
p^{old}	4.5	5.0
x^{new}	4.0	0.5
x^{old}	6.0	5.5

Table 2: Numerical example: $\alpha = 5$, $(a, b) = (1, 10)$

to understand the monopoly market in which old and new products coexist. The new product cannot survive when the highest marginal willingness to pay for the new product, b, is sufficiently low ($b < \frac{3}{2}\alpha$), and the old will exit when b is sufficiently high ($b > \frac{13}{2}\alpha$).

From the results, we find dynamic price changes; while the price of the new product *falls* in the second period, the price of the old product *rises*. Although it would be reasonable to set a higher price for the new product because it is durable, notably, the price of the old product rises, that is, the firm sets a lower price in the first period. Table 1 shows that the price rise occurs just when the marginal willingness to pay for the new product is not too low, $b \geq \frac{3}{2}\alpha$. The old product will face declining demand due to the introduction of a substitutable new product. The usual supply and demand analysis when a market is perfectly competitive and the leftward shift of the demand curve represents declining demand, could not explain this price rise.

Why does the firm lower the price of the old product? A possible explanation is the market expansion effect. A lower price makes the old product more attractive to consumers. Some consumers buy the old product in the first period, and they will purchase either the old or the new product in the second period.

In the numerical example, although all consumers ($b/a = 10$ consumers) buy something in the first period, 60% remain in this market in the second period. If the firm sets a higher price for the old product in the first period, many consumers buy the new product in this period and will buy nothing in the second period. The firm could obtain more profits by letting consumers purchase in both periods.

4 Concluding remarks

Firms frequently introduce new products into markets. Inevitably, some firms release new products while still producing substitutable products. This study investigates how a firm sets the price for existing products in such a situation. We construct a two-period durable goods monopoly model (only new product is durable), and demonstrate that the firm sets a low price for the old product in the first period and raises it later. This is profitable because the market expands; a customer purchases two products in total (the old product in the first period and the new product in the second).

Previous studies such as Ota (2011, 2019) and Yano et al. (2012, 2017) consider imperfectly competitive markets with an exogenously introduced new product. This study allows the monopoly firm producing the old product to set a price for the new product by itself. Therefore, the firm has the power to control the degree of decline in the demand for the old product. Endogenizing the degree of decline demand in the model is one of our contributions to the literature.

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