A Game Theoretical Study of "Duverger's Law"

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Abstract

"Duverger's Law" will be studied using a game theoretical model. The main result is two theorems. The Negative Theorem is that exit from an election, which means implicit alliance, occurs only if the Condorcet winner is not the biggest party. The Positive Theorem is that except for the case of a chicken game without suitable focal point, the Condorcet winner always wins the election in the equilibrium.

I want to thank John Guyton, Dennis Mueller and Mancur Olson for helpful comments.

1. Introduction

Duverger's law, which is usually understood to say that single-member-district plurality voting systems favor the two-party system, might be the most famous and disputed "law" in political science. The disputes have continued for at least 40 years after Duverger's "classic" (1951) or we should say it started before Duverger. (See Riker (1982, 1986) for the history.)

Duverger (1951, 1986) himself and other people, including Riker (1982, 1986) and Sartori (1976, 1986), were concerned with the number of parties. But we may be able to say that formal theorists of "Duverger's law" talk mainly about the number of candidates in each district and not about the number of parties itself. So in our discussion, we primarily talk about the number of candidates in each single-member-district under the plurality voting system.

One way of explaining "Duverger's law" is to study entry and entry deterrence in a position selecting model. Brams and Straffin (1982), Palfrey (1984), Greenberg and Shepsle (1987), Shepsle and Cohen (1990), Feddersen, Sened and Wright (1990), and Weber (1990, Forthcoming) are examples of this type of approach. It would be suitable for a presidential system like the U. S. where each candidate for congress can freely choose his or her position just for his or her own victory. But it might not be suitable for a parliamentary system where a party leader must decide the party's position for forming a cabinet and its candidate in each district is constrained by it. Since the countries where the applicability of Duverger's law is doubtful[1] use a parliamentary system, we might have to adopt other types of models to study Duverger's law.

One popular way is assuming some kind of voter's rationality and using insincere voting to explain the situation.

Riker (1976) introduced the notion of sophisticated voting and disillusioned voting, and analyzed the dynamics of the number of political parties. His model is very interesting but some of his assumptions might be a little bit arbitrary. This seems especially true where, in order to explain the Indian situation (one big party competes with many small parties), he assumed that the social axis of ideology restricts the direction of change of individuals' support.

Palfrey (1989) used sophisticated game theory but in his model multi-candidate competition like India is just a knife-edge case. A multi-party system should not be such a rare case.

Some voters vote insincerely in the real world. But is it the main reason of Duverger's law? It may not be a strong enough factor for the elimination of the third party. For example, in the Japanese plurality election (especially in the upper house general or the filling-vacancy election) usually JCP in addition to LDP and JSP has a candidate and continues to keep significant number of the votes. If the voters are fully rational the JCP supporters should vote for JSP.

A more important problem might be that candidates who know they will be defeated often do not exit from the election. Candidates must have more information and should be more rational than voters. If voters are rational, why do not candidates who will be defeated exit?

The exit of candidates has not been studied so much even though many people, including Riker (1982:761, 1986:33) and Shepsle (1991: 62) mentioned it along with insincere voting. One exception I know is Humes (1990). But he just gives examples and his model might not be perfectly game theoretical. This seems to be especially the case regarding his assumption number six, "if a political party is going to lose the next election with certainty, it prefers not expending effort of the election (withdrawing) to expending effort on the election". We should always have only one candidate except the knife-edge case because in his model voter's distribution is common knowledge.

Exit would mean implicit alliance rather than not expending effort. So it might not always occur. Exit might need some conditions. Let us study this in our game theoretical model.

2. Model

Let us set up our model. Our focal point is a local district, and the game is played there.

The Setting

1. The policy space is one-dimensional.

This might be a traditional assumption for simplicity and many position selecting type models adopt it. It might not be so unrealistic, because at least five main Japanese parties, LDP, DSP, Komei, JSP, JCP can be considered on one line. (See Iwai (1988:113).)

2. The election rule is Single-member-district plurality where a candidate wins an election if and only if he or she gains more votes than any other candidate. For convenience, it is assumed that ties do not occur.

Candidates

1. There are three nationwide parties, A, B and C. Each party's position is nationally decided and its

candidate in each district must have the same position. For simplicity no two parties share the same position in the policy space.

Three party models are used by many formal theorists, including Riker (1976), Palfrey (1989) and Humes (1990). It may be very natural to start to study multi-party systems from the simplest assumption about them.

2. A candidate in each district has two strategies, to run or to exit.
3. The cost of running is assumed to be 0.

Our results will hold for a small negative cost (or a small benefit) of running, but for simplicity we assume there is no cost or benefit. Contrary to the assumptions of some other authors, including Riker (1982:761, 1986:33) and Shepsle (1991:62), we are assuming that running would be beneficial for the party because its advertisement and total cost might be 0. This may be true especially if there were public support for the election. This assumption is supported by the fact that in Japan, in order to reduce the number of fly-by-night candidates, the candidates are asked to deposit some money. The fact that there is need to increase the deposit cost suggests that there is either no cost or actual positive benefit to a candidate for running.

4. Each candidate has single peaked preference. The payoff of the candidate depends on the final winner's position.

It may be a very natural assumption that Conservatives prefer

Socialists winning to Communists winning and Communists prefer Socialists winning to Conservatives winning. You may understand that (implicit) alliance between A and B or B and C are possible but A and C is impossible.

Voters

1. Each voter has single peaked preference.
2. Voters always vote sincerely.

With this assumption voters are not players in our game.

Solution

1. Trembling hand perfect equilibria are our solutions.

To assume trembling hand perfect equilibria means to assume there would be a small possibility that players make mistakes. Do political parties make some mistakes? Sure, they do. For example in Japanese multi-member district election LDP often has more candidates than the number of seats allocated to a particular district and JSP sometimes fails to have a candidate even though there is considered to be a significant chance for them to win. If you do not want to use trembling hand perfect equilibria, assume that there are small total benefits for candidates to run. In such cases Nash equilibria are enough to get the same solutions.

Model Solving

Because of the assumption of single peakedness of the voters, voters can be divided into four groups depending on their preference. (For simplicity, let us assume there is no indifferent case.)

group A A>B>C
group Ba B>A>C
group Bc B>C>A
group C C>B>A

Let us call the number of voters in each group #A, #Ba, #Bc and #C and also #B=#Ba+#Bc. According to the combination of the numbers our game is divided into only eight cases. Because of our assumptions about voters, the first inequality shows who would win the election between three, the second shows who would win the election between A and B, the third shows who would win the election between A and C, and the last shows who would win the election between B and C.

Case 1. (#A>#B,#C: #A>#B+#C; #A+#Ba>#Bc+#C; #A+#B>#C)
Case 2. (#A>#B,#C: #A<#B+#C; #A+#Ba>#Bc+#C; #A+#B>#C)
Case 3. (#A>#B,#C: #A<#B+#C; #A+#Ba<#Bc+#C; #A+#B>#C)
Case 4. (#B>#A,#C: #A<#B+#C; #A+#Ba>#Bc+#C; #A+#B>#C)
Case 5. (#B>#A,#C: #A<#B+#C; #A+#Ba<#Bc+#C; #A+#B>#C)
Case 6. (#C>#A,#B: #A<#B+#C; #A+#Ba>#Bc+#C; #A+#B>#C)
Case 7. (#C>#A,#B: #A<#B+#C; #A+#Ba<#Bc+#C; #A+#B>#C)
Case 8. (#C>#A,#B: #A<#B+#C; #A+#Ba<#Bc+#C; #A+#B<#C)

Because of the assumption of single peakedness of the candidate the game is solved as the tables.

Table 1.

Case 1. (#A>#B,#C; #A>#B+#C; #A+#Ba>#Bc+#C; ; #A+#B>#C)

A = Biggest = Condorcet Winner = A (A=Dominant)

A	B	C	Winner	
Run	**Run**	**Run**	**A**	Nash Trembling hand
Run	Run	Exit *	A	Nash
Run	Exit *	Run	A	Nash
Run	Exit *	Exit *	A	Nash
Exit X	Run	Run	B	
Exit X	Run	Exit	B	
Exit X	Exit X	Run	C	
Exit X	Exit X	Exit X	X	

Since the strategies with "X" are not optimal strategies for the candidate given other candidates' strategies, the sets of the strategies without "X" are Nash equilibria in pure strategy. The Nash equilibrium strategies with " * " are not optimal strategies for the candidate in the perturbed games. In this model it can be proved that the set of the strategies without " * " is the (unique) trembling hand perfect equilibrium.

Table 2.

Case 2. (#A>#B,#C; #A<#B+#C; #A+#Ba>#Bc+#C; ; #A+#B>#C)

A = Biggest ≠ Condorcet Winner = B

A	B	C	Winner	
Run	Run	Run X	A	
Run	**Run**	Exit	**B**	Nash Trembling hand
Run	Exit *	Run	A	Nash
Run	Exit X	Exit	A	
Exit X	Run	Run	B	
Exit *	Run	Exit	B	Nash
Exit X	Exit X	Run	C	
Exit X	Exit X	Exit X	X	

Table 3.

Case 3. (#A>#B,#C; #A<#B+#C; #A+#Ba<#Bc+#C; ; #A+#B>#C)

A = Biggest ≠ Condorcet Winner = B

Case 3-1. (B prefers A to C)

A	B	C	Winner	
Run	Run	Run X	A	
Run	**Run**	Exit	**B**	Nash Trembling hand
Run	Exit X	Run	C	
Run	Exit X	Exit X	A	
Exit X	Run	Run	B	
Exit *	Run	Exit	B	Nash
Exit	Exit X	Run	C	
Exit X	Exit X	Exit X	X	

Table 4.

Case 3. (#A>#B,#C; #A<#B+#C; #A+#Ba<#Bc+#C; ; #A+#B>#C)

A = Biggest ≠ Condorcet Winner = B

Case 3-2. (B prefers C to A)

A	B	C	Winner	
Run	Run X	Run X	A	Chicken ↑
Run	**Run**	Exit	**B**	Nash Trembling hand
Run	Exit	**Run**	**C**	Nash Trembling hand
Run	Exit X	Exit X	A	
Exit X	Run	Run	B	
Exit *	Run	Exit	B	Nash
Exit	Exit X	Run	C	
Exit X	Exit X	Exit X	X	

Table 5.

Case 4. (#B>#A,#C; #A<#B+#C; #A+#Ba>#Bc+#C; ; #A+#B>#C)

B = Biggest = Condorcet Winner = B

A	B	C	Winner	
Run	**Run**	**Run**	**B**	Nash Trembling hand
Run	Run	Exit *	B	Nash
Run	Exit X	Run	A	
Run	Exit X	Exit	A	
Exit *	Run	Run	B	Nash
Exit *	Run	Exit *	B	Nash
Exit X	Exit X	Run	C	
Exit X	Exit X	Exit X	X	

Table 6.

Case 5. (#B>#A, #C; #A<#B+#C; #A+#Ba<#Bc+#C; ; #A+#B>#C)

B = Biggest = Condorcet Winner = B

A	B	C	Winner	
Run	**Run**	**Run**	**B**	Nash Trembling hand
Run	Run	Exit *	B	Nash
Run	Exit X	Run	C	
Run	Exit X	Exit X	A	
Exit *	Run	Run	B	Nash
Exit *	Run	Exit *	B	Nash
Exit	Exit X	Run	C	
Exit X	Exit X	Exit X	X	

Table 7.

Case 6. (#C>#A, #B; #A<#B+#C; #A+#Ba>#Bc+#C; ; #A+#B>#C)

C = Biggest ≠ Condorcet Winner = B

Case 6-1. (B prefers A to C)

A	B	C	Winner	
Run X	Run X	Run	C	Chiken
Run	Run	Exit X	B	↑
Run	Exit	**Run**	**A**	Nash Trembling hand
Run	Exit X	Exit	A	
Exit	**Run**	**Run**	**B**	Nash Trembling hand
Exit	Run	Exit *	B	Nash
Exit X	Exit X	Run	C	
Exit X	Exit X	Exit X	X	

Table 8.

Case 6. (#C>#A,#B; #A<#B+#C; #A+#Ba>#Bc+#C; ; #A+#B>#C)

C = Biggest ≠ Condorcet Winner = B

Case 6-2. (B prefers C to A)

A	B	C	Winner	
Run X	Run	Run	C	
Run	Run	Exit X	B	
Run	Exit X	Run	A	
Run	Exit X	Exit	A	
Exit	**Run**	**Run**	**B**	Nash Trembling hand
Exit	Run	Exit *	B	Nash
Exit X	Exit X	Run	C	
Exit X	Exit X	Exit X	X	

Table 9.

Case 7. (#C>#A,#B; #A<#B+#C; #A+#Ba<#Bc+#C; ; #A+#B>#C)

C = Biggest ≠ Condorcet Winner = B

A	B	C	Winner	
Run X	Run	Run	C	
Run	Run	Exit X	B	
Run	Exit *	Run	C	Nash
Run	Exit X	Exit X	A	
Exit	**Run**	**Run**	**B**	Nash Trembling hand
Exit	Run	Exit *	B	Nash
Exit	Exit X	Run	C	
Exit X	Exit X	Exit X	X	

Table 10.

Case 8. (#C>#A,#B; #A<#B+#C; #A+#Ba<#Bc+#C; ; #A+#B<#C)

C = Biggest = Condorcet Winner = C (C=Dominant)

A	B	C	Winner	
Run	**Run**	**Run**	**C**	Nash Trembling hand
Run	Run	Exit X	B	
Run	Exit *	Run	C	Nash
Run	Exit X	Exit X	A	
Exit *	Run	Run	C	Nash
Exit	Run	Exit X	B	
Exit *	Exit *	Run	C	Nash
Exit X	Exit X	Exit X	X	

The intuition behind each case would be as follows. In case 1, since A is dominant, both B and C run for getting a rare chance at A's mistake. In case 2, in order to beat A who has the most support, C who has no chance if B runs is implicitly forced to exit to cooperate for B's winning. (B's winning is the second best choice for C next to him or herself because of the single peaked preference assumption of the candidate.) In case 3-1, since B prefers C to A, B can run anyway. C will be implicitly forced to cooperate because of his or her single peaked preference. In case 3-2, without a suitable focal point, the crash of the chicken game might occur. But the Condorcet winner may become a focal point. In case 4, since B is too strong, both A and C run for getting a rare chance to win. Cases 5, 6-1, 6-2, 7 and 8 are the mirror images of cases 4, 3-2, 3-1, 2 and 1, respectively.

We get two theorems from the table.

Negative Theorem

Exit, which means implicit alliance, occurs only if the Condorcet winner is not the biggest party.

Positive Theorem

Except for the case of a chicken game without suitable focal point, the Condorcet winner always wins the election in the equilibrium[2].

3. Concluding Remarks

To explain the Indian case where multi-party competition continues, Riker (1976) shows in his three party model that, for three parties to survive, the center party must be the largest[3]. Interestingly enough, our case 4 or 5 fits Riker's condition and result even though the mechanics are different.

According to our model, we can say that even though we chose a single-member-district plurality system, if we had used a parliamentary system where independence of each candidate is difficult, it might be tough to get a two party system. In Japan the government proposed an election system where about half of the members are chosen by a single-member-district plurality system and the rest are chosen separately by a proportional representation system. If Japan decided to use such an election system, LDP would win almost all single-member-districts. Since proportional representation parts reinforce the benefits to run for single-member-district part, the alliance of LDP oppositions would become much more difficult.

Our model might have many extensions. We implicitly assumed that all candidates know voter's distribution perfectly. It may not be realistic. The effect of imperfect information should be studied. We have also ignored national level politics. As Austen-Smith (1987) shows it is important. A two stage game should be studied.

Notes

1. Famous examples are Canada and India. Canada's three party system is usually explained by its strong geographical parties. India, where one big party and many small parties compete, is a disputable case. (See Riker (1976, 1982, 1986), Palfrey (1989), Humes (1990), etc.)
2. An election with run-off where top two candidates can run for the second election has a good character to escape from the possibility of the chicken game. If we change the second assumption of the setting to the election with run-off, our two theorems would become as following.

   Negative Theorem

   Exit, which means implicit alliance, occurs only if the Condorcet winner is the smallest party.

   Positive Theorem

   Condorcet winner becomes always the final winner.

   (Tables for the proof are available from the author.)
3. As Humes (1990:230) said, Riker (1982, 1986) might have stated his proposition incorrectly.

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